18. State and prove Eisenstein Criterion.

19. Let R be a Euclidean ring. Prove that any finitely generated R – module, M, is the direct sum of a finite number of cyclic submodules.

20. Prove the following.

   (a) $T \in A(v)$ is unitary if and only if $TT^* = 1$.

   (b) If T is Hermitian and $vT^k = 0$ for $k \geq 1$, then $vT = 0$.

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APRIL/MAY 2023

GMA11/DMA11 — ALGEBRA - I

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define normalizer of an element in a group.

2. Prove that a group of order 35 is cyclic.

3. Define internal direct product of normal subgroups of a group.

4. Let A be a group of order 3 and B be a group of order 5. Prove that $A \times B$ is a cyclic group.

5. Define irreducible polynomial over a field.

6. Prove that $x^2 + 1$ is irreducible over the integers mod 7.

7. Define R–module, where R is any ring.

8. Let $T \in A(V)$. Define index of nilpotenle of T.

9. Define unitary linear transformation.

10. Define Hermitian transformation.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that if $O(G) = p^2$, where P is a prime number, then G is abelian.

Or

(b) Prove that $n(K) = 1 + p + \dots\dots + p^{k-1}$, where $n(K)$ is defined by $p^{n(k)} / (p^k)!$ but $p^{n(k)+1} \nmid (p^k)$.

12. (a) Prove that $A \times B$ is isomorphic to $B \times A$ where A and B are groups.

Or

(b) Let G be a group and suppose that G is the internal direct product of normal subgroups $N_1, N_2, \dots\dots N_n$. Let $T = N_1 \times N_2 \times \dots \times Nn$. Prove that G and T are isomorphic.

13. (a) State and prove division algorithm.

Or

(b) State and prove Gauss lemmer.

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14. (a) If $T \in A(V)$ is nilpotent, prove that $\alpha_0 + \alpha_1 T + \dots . \alpha_m T^m$, where $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.

Or

(b) If $V = V_1 \oplus \dots . \oplus V_k$ where each V_i is invariant under T and if $p_1(x)$ is the minimal polynomial over F of T_i, the linear transformation induced by T on V_i, prove that the minimal polynomial of T over F is the least common multiple of $p_1(x), p_2(x), \dots\dots, p_k(x)$.

15. (a) If $(vT, vT) = (v, v)$ for all $v \in V$. Prove that T is unitary.

Or

(b) If N is normal and if $vN^k = 0$. Prove that $vN = 0$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Sylow's theorem.

17. Prove that every finite abelian group is the direct product of cyclic groups.